Esercitazione 02/05/'11

Oggi va parlerò e farò esercizi sul capitolo 10.

Il primo argomento sono gli integrali curvilinei

Def: Sia γ curva parametrizzata da

$$r : [a, b] \longrightarrow \mathbb{R}^3$$

$$\int_\gamma f \, ds = \int_a^b f(r(t)) \cdot \|r'(t)\| \, dt$$

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es.

$$r : t \longmapsto \begin{vmatrix} e^t \cos t \\ e^t \sin t \\ e^t \end{vmatrix} \qquad f\left(\begin{vmatrix} x \\ y \\ z \end{vmatrix}\right) = \ln z$$

$r : [0,1] \longrightarrow \mathbb{R}^3$

$$r'(t) = \begin{vmatrix} e^t \cdot \cos t + e^t \cdot \sin t \\ e^t \cdot \sin t - e^t \cos t \\ e^t \end{vmatrix}$$

$$\|r'(t)\| = e^{2t} \cos^2 t + e^{2t} \sin^2 t + \cancel{2 e^{2t} \cos t \sin t}$$
$$e^{2t} \sin^2 t + e^{2t} \cos^{2} t - \cancel{2 \cdot e^{2t} \cos t \sin t}$$
$$e^{2t}$$
$$= 3 \cdot e^{2t}$$

$$\int_\gamma \ln z \, ds = \int_0^1 \ln e^t \cdot \sqrt{3} \cdot e^t \, dt$$

$$= \sqrt{3} \int_0^1 t \cdot e^t \, dt = \sqrt{3} \, t \cdot e^t \Big|_0^1 - \sqrt{3} \int_0^1 e^t \, dt$$

$$= \sqrt{3}$$

Domanda: integrale dipende dalla parametrizzazione? ?

troviamo nuova parametrizzazione

$$s : [0, \pi/2] \longrightarrow \mathbb{R}^3$$

$$\theta \longmapsto \begin{pmatrix} e^{\sin\theta} \cos(\sin\theta) \\ e^{\sin\theta} \sin(\sin\theta) \\ e^{\sin\theta} \end{pmatrix}$$

$$s'(\theta) = \begin{pmatrix} e^{\sin\theta} \cos\theta \cdot \cos(\sin\theta) - e^{\sin\theta} \sin(\sin\theta) \cdot \cos\theta \\ e^{\sin\theta} \cos\theta \cdot \sin(\sin\theta) + e^{\sin\theta} \cos(\sin\theta) \cdot \cos\theta \\ e^{\sin\theta} \cos\theta \end{pmatrix}$$

$$\|s'(\theta)\|^2 = e^{2\sin\theta} \cos^2\theta \cdot \cos^2(\sin\theta) + e^{2\sin\theta} \cdot \sin^2(\sin\theta) \cdot \cos^2\theta$$
$$- 2 e^{2\sin\theta} \cos^2\theta \cdot \cos(\sin\theta) \cdot \sin(\sin\theta)$$
$$e^{2\sin\theta} \cos^2\theta \cdot \sin^2(\sin\theta) + e^{2\sin\theta} \cdot \cos^2(\sin\theta) \cdot \cos^2\theta$$
$$+ 2 e^{2\sin\theta} \cdot \sin(\sin\theta) \cdot \cos(\sin\theta)$$
$$+ e^{2\sin^2\theta} \cdot \cos^2\theta$$

$$\|s'(\theta)\| = \sqrt{3} \cdot e^{\sin\theta} \cdot \cos\theta$$

$$\int_\gamma \ln z \, dz = \int_0^{\pi/2} \sin\theta \cdot \cos\theta \cdot \sqrt{3} \cdot e^{\sin\theta} \, d\theta$$

$$= \sqrt{3} \int_0^{\pi/2} \sin\theta \cdot e^{\sin\theta} \cdot \cos\theta \, d\theta$$

$$t = \sin\theta \qquad dt = \cos\theta \, d\theta$$

$$= \sqrt{3} \int_0^1 t \cdot e^t \, dt$$

cioè, la m

del vettore n

!!! prende comunque n

cost. di parametrizzazion

$$\cancel{G(x,y)} = \begin{vmatrix} \dfrac{kQ\,x}{(x^2+y^2)^{3/2}} \\ \\ \dfrac{kQ\,y}{(x^2+y^2)^{3/2}} \end{vmatrix}$$

$$E(x,y) =$$

Calcoliamo lavoro lungo la curva $t \longmapsto \begin{vmatrix} t \\ t \end{vmatrix}$

$t \in [1,2]$

$$\cancel{kQ \int_1^2 \frac{x}{\sqrt{2}t}}$$

$$kQ \int_1^2 \left\langle \begin{vmatrix} \dfrac{t}{(\sqrt{2}\,t)^{3}} \\ \dfrac{t}{(\sqrt{2}\,t)^{3}} \end{vmatrix} , \begin{vmatrix} 1 \\ 1 \end{vmatrix} \right\rangle dt$$

$$kQ \int_1^2 2 \cdot \frac{1}{2\sqrt{2}} \frac{1}{t^2}\, dt = \frac{\sqrt{2}}{2} kQ \cdot -\frac{1}{t}\Big|_1^2$$

$$= \frac{\sqrt{2}}{4} kQ \qquad \text{OK!}$$

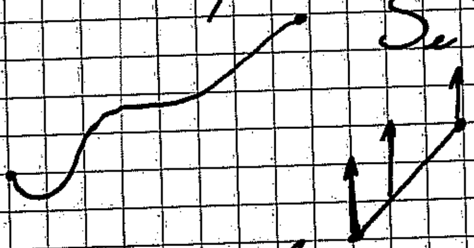
~~Su un altro percorso~~

$$\cancel{t \longmapsto \begin{vmatrix} t \\ t + \sin 2\pi t \end{vmatrix}}$$

$$\cancel{kQ \int_1^2 \left| \begin{matrix} \dfrac{t}{(t^2 + t^2 + 2t \sin 2\pi t + \sin^2 2\pi t)} \\ \\ \end{matrix} \right| \begin{vmatrix} 1 \\ 1 + 2\pi \cos 2\pi t \end{vmatrix}}$$

$$\cancel{kQ \int_1^2 \frac{t}{(t + \sin 2\pi t)}}$$

Integrali curvilinei di campi vettoriali
essenzialmente generalizza concetto di lavoro di una forza lungo curva.



Se sposta lungo una retta, forza costante, lavoro era $\vec{F}\cdot(\overrightarrow{r_2 - r_1})$.

Def: Il lavoro compiuto da F per spostare un punto lungo γ da $r(a)$ ad $r(b)$ è

$$\int_\gamma F\cdot T\, ds = \int_a^b F(r(t))\cdot r'(t)\, dt.$$

Se curva chiusa circuitazione di $\vec{F}$ lungo γ

$$\oint_\gamma F\cdot T\, ds$$

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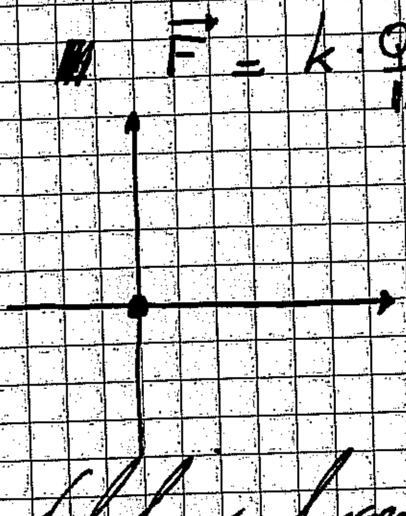
es. Campo elettrico generato da una carica isolata nell'origine.

$$\vec{F} = k\cdot\frac{Q\cdot q}{\|r\|^3}\cdot\vec{r} \qquad \frac{F}{q} = \vec{E} = \frac{k\cdot Q}{\|r\|^3}\vec{r} = \frac{c}{\|r\|^3}\cdot\vec{r}$$

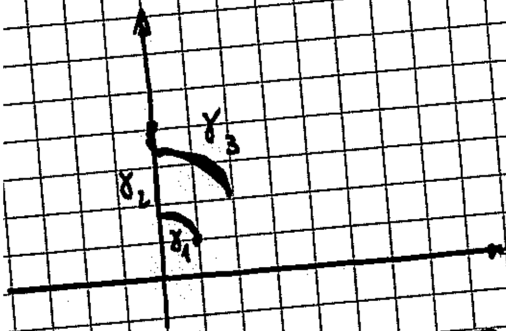


$$\cancel{E(r,\theta)=\begin{vmatrix}\frac{c}{r^{3/2}}\cdot\cos\theta\\ \frac{c}{r^2}\cdot\sin\theta\end{vmatrix}} \quad \cancel{E(x,y) = \left|\quad\right|}$$

Calcoliamo lavoro lungo la curva

~~kQ~~

Dai ora altro percorso



$$r_1 : \left[0, \tfrac{\pi}{4}\right] \longrightarrow \gamma_1$$

$$\theta \longmapsto \begin{pmatrix} \sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right) \\ \sqrt{2}\cdot\sin\left(\frac{\pi}{4}+\theta\right) \end{pmatrix}$$

$$\int_0^{\pi/4} \left\langle \begin{pmatrix} \dfrac{kQ\sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right)}{(2)^{3/2}} \\[2ex] \dfrac{kQ\sqrt{2}\sin\left(\frac{\pi}{4}+\theta\right)}{2^{3/2}} \end{pmatrix}, \begin{pmatrix} -\sqrt{2}\sin\left(\frac{\pi}{4}+\theta\right) \\ \sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right) \end{pmatrix} \right\rangle d\theta = 0$$

$$\frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

e ~~modo~~ così nello stesso lungo γ_3.

$$r_2 : \left[\sqrt{2}, 2\sqrt{2}\right] \longrightarrow \gamma_2$$

$$t \longmapsto \begin{pmatrix} 0 \\ t \end{pmatrix}$$

$$\int_{\sqrt{2}}^{2\sqrt{2}} \left\langle \begin{pmatrix} 0 \\ \dfrac{kQt}{t^3} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\rangle dt = -kQ\frac{1}{t}\Bigg|_{\sqrt{2}}^{2\sqrt{2}}$$

$$= kQ \cdot \frac{1}{2\sqrt{2}} =$$

Quindi lungo due percorsi diversi lo stesso risultato!

Lungo circuito chiuso, quindi è 0!!!

Es. $\cancel{B = \frac{\mu_0}{2\pi}\cdot\frac{i}{r^2}\cdot \pi r}$

Il campo magnetico

$$\vec{B} = \frac{\mu_0}{2\pi}\cdot\frac{i}{x^2+y^2}\begin{vmatrix} y \\ -x \end{vmatrix} \qquad \vartheta \longmapsto \begin{matrix}\cos\vartheta \\ \sin\vartheta\end{matrix}$$

$$\cancel{\int_0^{2\pi} \frac{\mu_0}{2\pi}\cdot\frac{1}{r}}$$

$$\frac{\mu_0\cdot i}{2\pi}\int_0^{2\pi} \left\langle \begin{vmatrix}\sin\vartheta \\ -\cos\vartheta\end{vmatrix}, \begin{vmatrix}\sin\vartheta \\ -\cos\vartheta\end{vmatrix} \right\rangle d\vartheta = \mu_0\cdot i$$

Questo ha integrale non nullo lungo curve chiuse.

Def. Un campo è detto conservativo se è il gradiente di una qualche funzione, cioè esiste $U : \mathbb{R}^2(\mathbb{R}^3) \longrightarrow \mathbb{R}$ t.c.

$$\vec{F} = \nabla U$$

es. C. E.

$$\vec{E} = \begin{vmatrix} \dfrac{kQ\,x}{(x^2+y^2)^{3/2}} \\[2ex] \dfrac{kQ\,y}{(x^2+y^2)^{3/2}} \end{vmatrix} \qquad U = \frac{kQ}{(x^2+y^2)^{1/2}} + C$$

$$\frac{\partial U}{\partial x} = \frac{1}{\cancel{2}}\,\frac{kQ}{(x^2+y^2)^{3/2}}\cdot \cancel{2}x = \frac{kQx}{(x^2+y^2)^{3/2}} \quad \text{Ok.}$$

Th. $F: A \to \mathbb{R}^2$ c. v. di classe C^1

TFAE

① F conservativo

② se γ_1, γ_2 contenute in A con gli stessi estremi

$$\int_{\gamma_1} F \cdot T \, ds = \int_{\gamma_2} F \cdot T \, ds$$

③ curva chiusa γ contenuta in A

$$\oint F \cdot T \, ds = 0$$

Cond. necessaria per conservativo in $\mathbb{R}^2$

Se $F = \nabla U$ $\qquad \vec{F} = \vec{F_1} \cdot \vec{i} + \vec{F_2} \cdot \vec{j}$

$$F_1 = \frac{\partial U}{\partial x} \qquad F_2 = \frac{\partial U}{\partial y}$$

$$\frac{\partial F_1}{\partial y} = \frac{\partial U}{\partial x \partial y} \qquad \frac{\partial F_2}{\partial x} = \frac{\partial U}{\partial x \partial y}$$

cond. necessaria perché campo conservativo in $\mathbb{R}^2$ è che der. in croce sono uguali.

Se F è campo in $\mathbb{R}^3$ condizione necessaria è che rot $F = 0$

$$\nabla \wedge F = 0 \qquad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

Calcoliamo il rotore di

$$\frac{1}{(x^2+y^2+z^2)^{3/2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$ il campo elettrico

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{(x^2+y^2+z^2)^{3/2}} & \frac{y}{(x^2+y^2+z^2)^{3/2}} & \frac{z}{(x^2+y^2+z^2)^{3/2}} \end{vmatrix}$$

$$\vec{i}\left(\frac{3zy}{(x^2+y^2+z^2)^{5/2}} - \frac{3zy}{(x^2+y^2+z^2)^{5/2}}\right)$$

$$+\vec{j} \quad \ldots$$

$$+\vec{k} \quad \ldots$$

Quindi è irrotazionale

Importante: campo irrotazionale ~~conservativo~~, dominio semplicemente connesso

$\Longrightarrow$ conservativo!

Teor. di Gauss-Green

$D \subseteq \mathbb{R}^2$. $F(x,y)$ campo C^1 in aperto che contiene D. Allora

$$\iint_D (\partial_x F_2 - \partial_y F_1)\,dx\,dy = \oint_{\partial D^+} F \cdot T\,ds$$

Calcoliamo l'integrale

$$F = \begin{vmatrix} 2y + \cos(e^x) \cdot \sin(e^x) \\ x + \sin(e^y + y^3) \end{vmatrix}$$ è C^1 su aperti che contiene cerchio!

$$\oint_\gamma F \cdot T \, ds$$ $\gamma: [0, 2\pi] \longrightarrow \gamma$

$\vartheta \longmapsto \begin{vmatrix} \cos\vartheta \\ \sin\vartheta \end{vmatrix}$

$$\oint_\gamma F \cdot T \, ds = \iint_D \left(\frac{\partial}{\partial x} F_2 - \frac{\partial}{\partial y} F_1 \right) dx\, dy$$

$$= \iint_D (1 - 2)\, dx\, dy = -\iint_D dx\, dy$$

è l'area del cerchio di raggio 1 e quindi

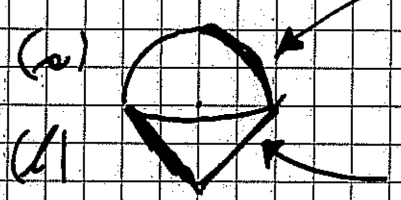
$$\oint_\gamma F \cdot T \, ds = -\iint_D 1 \, dx\, dy = -\pi \;!!!$$

Integrali di superficie

Def: $f : A \subset \mathbb{R}^3 \longrightarrow \mathbb{R}$ superficie regolare di bordo Σ $\quad r : D \longrightarrow \mathbb{R}^3$ (par.)

$$\iint_\Sigma f \, d\sigma = \iint_D f(r(u,v)) \, \| r_u(u,v) \times r_v(u,v) \| \, du\,dv$$

Es. calcoliamo la massa della lamina



$$\begin{cases} x^2 + y^2 + z^2 = 1 \\ z \geq 0 \end{cases}$$

$$\begin{cases} z = x^2 + y^2 - 1 \\ -1 \leq z \leq 0 \end{cases}$$

$$f(x,y,z) = x^2 + y^2$$

(a) $\quad r_1 \quad \begin{vmatrix} \vartheta \\ \varphi \end{vmatrix} \longmapsto \begin{vmatrix} \cos\vartheta \cos\varphi \\ \sin\vartheta \cos\varphi \\ \sin\varphi \end{vmatrix} \qquad \vartheta \in [0, 2\pi] \quad \varphi \in [0, \pi/2]$

$$r_{1,\vartheta} = \begin{vmatrix} -\sin\vartheta \cos\varphi \\ \cos\vartheta \cos\varphi \\ 0 \end{vmatrix} \qquad r_{1,\varphi} = \begin{vmatrix} -\cos\vartheta \sin\varphi \\ -\sin\vartheta \sin\varphi \\ \cos\varphi \end{vmatrix}$$

$$\begin{vmatrix} i & j & k \\ -\sin\vartheta\cos\varphi & \cos\vartheta\cos\varphi & 0 \\ -\cos\vartheta \sin\varphi & -\sin\vartheta\sin\varphi & \cos\varphi \end{vmatrix}$$

$\vec{i}$ $\vec{j}$ $\vec{k}$

$\cos\theta$ $\sin\theta$ $2r$

$-r\sin\theta$ $r\cos\theta$ 0

$\vec{i}\left(2r^2\cos\theta\right)+\vec{j}\left(+2r^2\sin\theta\right)+\vec{k}\; r$

$\| r_\theta \wedge r_\rho \| = \sqrt{4r^4 + r^2}$

$= r\sqrt{4r^2+1}$

$\int_0^{2\pi}\int_0^1 r^2 \cdot r\sqrt{4r^2+1}\; dr\, d\theta$

$\int f' \cdot g = f \cdot g - \int f \cdot g'$

$f' = r \cdot \sqrt{4r^2+1}$ $\quad g = r^2$

~~$f = \frac{1}{12}\sqrt{(4r^2+1)^{3/2}}$~~

$\int \sqrt{4r^2+1}\; r\, dr$

$t = 4r^2 \quad dt = 8r\,dr$

$\frac{1}{8}\int \sqrt{t+1}\; dt$

$\frac{1}{12}(t+1)^{3/2}$

$\frac{1}{12}\left(4r^2+1\right)^{3/2}$

$2\pi \left. \frac{r^2}{12}(4r^2+1)^{3/2} \right|_0^1 - \int_0^1 \frac{1}{6}(4r^2+1)^{3/2}\, r\, dr$

$2\pi \cdot \frac{1}{12}(4+1)^{3/2} -$

$\int \frac{1}{6}(4r^2+1)^{3/2} \cdot r\, dr$

$t = 4r^2$

$dt = 8r\,dr$

$= \frac{1}{48}\int (t+1)^{3/2}\, dt$

$= \frac{1}{48} \cdot \frac{2}{5}(t+1)^{5/2}$

$= \frac{1}{24} \cdot \frac{1}{5}(t+1)^{5/2} \Big|_0^1$

$\vec{i} . \cos\vartheta \cos^2\varphi - \vec{j} . (-\sin\vartheta \cos^2\varphi)$

$+ \vec{k} (\sin^2\vartheta \cos\varphi \sin\varphi + \cos^2\vartheta \cos\varphi \sin\varphi)$

$$r_\vartheta \wedge r_\varphi = \cos\varphi \begin{vmatrix} \cos\vartheta \cos\varphi \\ \sin\vartheta \cos\varphi \\ \sin\varphi \end{vmatrix}$$

$$\| r_\vartheta \wedge r_\varphi \| = \cos\varphi$$

$$\int_0^{2\pi} \int_0^{\pi/2} \cos\varphi \left(\cos^2\vartheta \cos^2\varphi + \sin^2\vartheta \cos^2\varphi \right) d\vartheta\, d\varphi$$

$$= \int_0^{2\pi} \int_0^{\pi/2} \cos^3\varphi \, d\vartheta\, d\varphi$$

$\cancel{\sin x - \frac{\sin(x)^3}{3}}$

$$= \int_0^{2\pi} \int_0^{\pi/2} \cos\varphi (1 - \sin^2\varphi) \, d\vartheta\, d\varphi$$

$$= \int_0^{2\pi} \int_0^{\pi/2} \cos\varphi - \sin^2\varphi \cos\varphi \, d\varphi\, d\vartheta$$

$$= 2\pi \left(\sin\varphi - \frac{\sin^3\varphi}{3} \Big|_0^{\pi/2} \right) = 2\pi \left(1 - \frac{1}{3} \right) = \frac{4}{3}\pi$$

$\ell)$ $r_2 \begin{vmatrix} r \\ \vartheta \end{vmatrix} \longrightarrow \begin{vmatrix} r \cos\vartheta \\ r \sin\vartheta \\ -1 + r^2 \end{vmatrix}$

$$r_{2,r} = \begin{vmatrix} \cos\vartheta \\ \sin\vartheta \\ +2r \end{vmatrix} \qquad r_{2,\vartheta} = \begin{vmatrix} -r \sin\vartheta \\ +r \cos\vartheta \\ 0 \end{vmatrix}$$

Teor. rotore:

- Sia Σ sup. semplice, regolare fino al bordo, orientabile, param. da $r: D \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^3$.

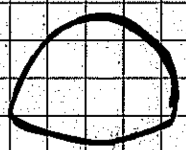
$N = r_u \wedge r_v / \| r_u \wedge r_v \|$, frontiera or. positivamente $\partial^+ \Sigma$

- $F: A \subset \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ c.v. C^1

allora

$$\iint_\Sigma \operatorname{rot} F \cdot N = \oint_{\partial^+\Sigma} F \cdot T \, ds$$

Es. calotta sferica



$$\Sigma = \begin{cases} x^2 + y^2 + z^2 = 1 \\ z \geq 0 \end{cases}$$

$$\partial \Sigma = \{ x^2 + y^2 = 1 \, , \, z = 0 \}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & 3x & y^2 \end{vmatrix} \quad \operatorname{rot} F = \vec{i} \cdot 2y + \vec{j} + 3\vec{k}$$

$$\vec{F} = \begin{vmatrix} z \\ 3x \\ y^2 \end{vmatrix}$$

$$\frac{2\pi}{12} \cdot 5^{3/2} - \frac{1}{120} \cdot 2^{5/2} + \frac{1}{120} \qquad \text{Ok!}$$

Def. Campo vett. continuo $\bar F : A \longrightarrow \mathbb{R}^3$, Σ sup regolare, param $r : D \longrightarrow \mathbb{R}^3$ il flusso

$$\iint_\Sigma F \cdot N \, d\sigma = \iint_D F \cdot r_u \wedge r_v \, du \, dv$$

Es. C. E. carica nell'origine

$$\begin{vmatrix} \dfrac{x}{(x^2+y^2+z^2)^{3/2}} \\ \dfrac{y}{(x^2+y^2+z^2)^{3/2}} \\ \dfrac{z}{(x^2+y^2+z^2)^{3/2}} \end{vmatrix}$$

$$\begin{vmatrix} \theta \\ \varphi \end{vmatrix} \longmapsto \begin{vmatrix} \cos\theta \cos\varphi \\ \sin\theta \cos\varphi \\ \sin\varphi \end{vmatrix} \qquad r_\theta \wedge r_\varphi = \cos\varphi \begin{vmatrix} \cos\theta \cos\varphi \\ \sin\theta \cos\varphi \\ \sin\varphi \end{vmatrix}$$

$$\int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \left\langle \begin{vmatrix} \cos\theta \cos\varphi \\ \sin\theta \cos\varphi \\ \sin\varphi \end{vmatrix}, \cos\varphi \begin{vmatrix} \cos\theta \cos\varphi \\ \sin\theta \cos\varphi \\ \sin\varphi \end{vmatrix} \right\rangle d\theta \, d\varphi$$

$$\int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \cos\varphi \, d\varphi \, d\theta = 2\pi \cdot \sin\varphi \Big|_{-\pi/2}^{\pi/2} = 4\pi$$

$$\iint_{\Sigma} \begin{vmatrix} 2y \\ 1 \\ 9x^2 \end{vmatrix} \cdot N \, d\sigma$$

$$\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} \left\langle \begin{vmatrix} 2\sin\vartheta\cos\varphi \\ 1 \\ 3\cancel{\cos^2\vartheta\cos^2\varphi} \end{vmatrix}, \begin{vmatrix} \cos\vartheta\cos\varphi \\ \sin\vartheta\cos\varphi \\ \sin\varphi \end{vmatrix} \right\rangle d\vartheta \, d\varphi$$

$$= \int_0^{2\pi} \left\langle \begin{vmatrix} 0 \\ 3\cos\Psi \\ \sin^2\Psi \end{vmatrix}, \begin{vmatrix} -\sin\Psi \\ \cos\Psi \\ 0 \end{vmatrix} \right\rangle d\Psi$$

$$= \int_0^{2\pi} 3\cos^2\Psi \, d\Psi = 3\int_0^{2\pi} \cos^2\Psi \, d\Psi$$

$$\cos 2\Psi = \cos^2\Psi - \sin^2\Psi$$
$$= \cos^2\Psi - (1 - \cos^2\Psi)$$
$$= 2\cos^2\Psi - 1$$

$$\cos^2\Psi = \frac{\cos 2\Psi + 1}{2}$$

$$3\int_0^{2\pi} \frac{\cos 2\Psi + 1}{2} \, d\Psi = \frac{3\Psi}{2}\Big|_0^{2\pi} + \frac{\sin 2\Psi}{4}\Big|_0^{2\pi}$$

$$= 3\pi \quad !!!$$

Teor. divergenza

- $E \subseteq \mathbb{R}^3$ regione solida, ∂E sup. regolare, norm ∂E
- $F : A \subset \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ c.v. C^1, A aperto
  $E \subseteq A$

$$\iiint_E \operatorname{div} F \, dx\, dy\, dz = \iint_{\partial E} F \cdot N_e \, d\sigma$$

Es. calcolare il flusso di

$$\vec{F} = \begin{pmatrix} 4z + 5y^2 \\ e^x + 6e^z \\ y^3 + e^{4x} \end{pmatrix} \quad \text{attraverso} \quad x^2 + y^2 + z^2 = 1$$

ora, la prima cosa che bisogna controllare è div F

$$\operatorname{div} F = 0 \;!!!$$

Quindi

$$\iint_{\partial E} \vec{F} \cdot \vec{N} \, d\sigma = \iiint_E \operatorname{div} F = 0 \;!!!$$